CONSTITUTIVE LAWS FOR SEA ICE DYNAMICS MODELS

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LONG TERM GOALS

The long term goal of my research is to develop sea ice dynamics models that describe behavior on kilometer scales and larger, and to base these models on the smaller scale physical processes known to control leading, rafting, and ridging.

OBJECTIVES

Three objectives were (1) to continue development of the new anisotropic plasticity constitutive law for large scale sea ice behavior, (2) to analyze data from controlled-load tests conducted during SIMI, and (3) to formulate constitutive laws and solution methods a heterogeneous multiscale material using micromechanics of random media.

APPROACH

We describe the presence of individual ridges, rafts, and leads in larger scale models by describing ice behavior as an anisotropic elastic plastic material. An oriented thickness distribution describes ice conditions. We assign each new lead system its own thickness distribution, which describes thermal growth and mechanical redistribution as the lead evolves over time. Ridges result from closing these lead systems. The surrounding older, thicker is assumed to be isotropic, and its strength changes only a little.

The new anisotropic constitutive law consists of a yield surface, flow rule, elastic closure, and kinematic relationship. The yield surface, which is described in stress component space (three Cartesian components, not two invariants as with an isotropic model), is composed of separate surfaces for each lead or ridge system. Each new lead introduces another oriented yield surface. A new lead may form only when the traction across it is zero. Ice conditions are described by an oriented thickness distribution. Both of these parts of the constitutive law were presented by *Coon, et al.* [1997a,b], which focuses on describing the physical behavior of ice and relating it to behavior of the anisotropic model. For example, we consider how to satisfy the requirement that a lead may

not open when there is a confining stress across it. This requirement leads to restrictions in yield surface shape, and a need to consider a non-normal flow rule.

Pritchard [1997] presented a more formal set of equations, where the ice condition is described by a three-dimensional oriented thickness distribution, rather than a thickness distribution for each lead or ridge system. This formulation allows a more consistent definition of ice strength in different directions.

We analyzed data from a set of SIMI tests in which we towed ice blocks across the ice sheet. The force balance equation describing this behavior has been derived and solved. Measured force histories are compared with modeled histories.

A spatially random granular Mohr-Coulomb material is obtained from the classical, homogeneous continuum model by taking the internal friction angle and the resistance in tension as a vector random field. These two constitutive coefficients specify a meso-continuum approximation on a meso-scale corresponding to the actual choice of spacing of a finite difference net of characteristics employed in the solution of a given boundary value problem.

ACCOMPLISHMENTS

The model formulation is progressing. We have outlined the anisotropic constitutive law. Acceptable yield surfaces and flow rules are now available, as are the elastic closure and kinematic relationship. A three-dimensional thickness distribution describes the oriented ice. We must still define ice strength in terms of ice conditions, and test the model in *0d* and *2d* simulations.

Data from the controlled-load tow tests are being used to estimate friction coefficients in the new stick-slip model. A model of the pulling force has been introduced and solved. Results show a definite stick-slip behavior, which is similar to that seen during full-scale ice rafting and ridging events. The stick-slip behavior depends on the elasticity of the surrounding ice sheet. These results, along with the model, are described in a manuscript being prepared for JGR [*Pritchard, et al.*, 1997].

We have also learned that noise generated by this process is strongly correlated with the towing force. Thus, if we can relate the spring constant to some property of the towing force, we will be able to estimate the flexibility of the ice cover from noise observations during rafting events at South Camp.

Explicit equations have been derived for the determination of the two families of characteristics in a Mohr-Coulomb medium. This derivation is analogous, albeit more complicated, than an analogous derivation carried out for materials obeying the Huber-Mises yield condition (typically, metals); [*Ostoja-Starzewski*, 1992; *Ostoja-Starzewski and Ilies*, 1996].

Due to the spatial fluctuations in the granular ice field medium, the characteristics display perturbations in their evolution throughout the domain of dependence. Consequently, wedges of

diffusing characteristics replace the unique unperturbed characteristics of a deterministic (homogeneous) medium problem. The entire formulation falls into a general framework of micromechancally based stochastic finite element/difference methods, where a single finite element (or difference) plays the role of a bridge between the small scale variability and the macroscopic response [*Ostoja-Starzewski*, 1993].

A computer program has been developed to assess the influence of spatial material randomness on the plastic response of granular-type ice fields for a range of these two key parameters has been carried out.

HONORS

Dr. Pritchard was elected to the grade of Fellow of the American Society of Mechanical Engineers during 1997.

SCIENTIFIC/TECHNICAL RESULTS

The framework of a new anisotropic elastic plastic constitutive law for sea ice has been described and accepted for publication by JGR [*Coon, et al.*, 1997a]. The concept of an oriented thickness distribution has been completed and accepted for publication by JGR [*Coon, et al.*, 1997b]. A more complete mathematical formulation using a general three-dimensional thickness distribution function has been published [*Pritchard*, 1997].

A comparison has also been made of the effects of small scale material variability on Mohr-Coulomb (MC) type versus Huber-Mises (HM) type plastic media. For very weak noise there is only a small difference between the ensemble average net of slip-lines of the stochastic problem (i.e., for a random medium) and the net of a corresponding deterministic problem (i.e., for a homogeneous medium). This difference and the accompanying scatter increase as material parameter fluctuations grow. Materials governed by the MC yield criterion appear to be more sensitive to noise than those governed by the HM criterion. Finally, there are no differences between results of a forward or a backward integration method for the HM materials; but a backward difference is recommended for the MC media.

IMPACT FOR SCIENCE AND SYSTEMS APPLICATIONS

The anisotropic elastic-plastic constitutive law and the oriented ice thickness distribution are new concepts. They allow the ice behavior to be described on scales from as small as a few kilometers up to hundreds of kilometers. For the first time, we can describe the formation and evolution of individual lead and ridge systems, including their orientations.

The oriented thickness distribution is an important new tool for analyzing satellite images. The amount of open water determined by estimating deformations from the usual contour integral can cause large errors, but these errors can be eliminated by aligning coordinates with the lead

systems observed in the image.

Because of its ability to calculate open water fractions more accurately, the anisotropic plasticity model should provide a more effective climate dynamics model. Thus, by incorporating the smaller scale behavior, the model should describe large scale behavior more accurately. Future simulations should compare simulated open water fractions to observed values, in addition to comparing simulated and observed ice motions. This added comparison will likely make it harder to adjust a single strength parameter to improve simulations, as is often done now.

Ice forecasting products can add the directions of leads to the usual thickness categories. This variable can have an important impact on the usefulness of forecasts for ship navigation and for offshore terminal operations.

The anisotropic model will provide a major new capability for the Navy sea ice forecasting needs.

This study demonstrates a method for treatment of multiscale mechanical phenomena. The analysis carried out also illustrates the importance of small scale (ice floe length scales) random variability in material properties on macroscale response.

TRANSITIONS

Within the next few years, I expect the new anisotropic plasticity model to be incorporated into the PIPS ice forecasting system, which will provide the Navy with a next-generation model.

RELATED PROJECTS

This work is being conducted in close cooperation with Dr. Max Coon and colleagues at NorthWest Research Associates, Inc. The model development is a collaborative effort. We also conducted the suite of SIMI controlled-load tests together, and jointly analyzed the results of these tests.

The task titled *Multiscale Microdynamics of Ice Fields* was subcontracted to Dr. Martin Ostoja-Starzewski.

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